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SYNTHETIC SEISMOGRAMS IN LATERALLY HETEROGENEOUS ANELASTIC MEDIA: MODAL SUMMATION FOR THE CASE OF OFFSHORE SEISMIC SOURCES (DEEP-SEA TROUGH)

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ABSTRACT

While waiting for the increment of strong motion data, especially for earthquake prone areas outside the United States and Japan, a very useful approach to perform immediate site specific seismic hazard assessment is the development and use of modelling tools. They are based, on one hand, on the theoretical knowledge of the physics of the seismic source and of wave propagation and, on the other hand, on the exploitation of the relatively rich database, already available, that can be used for the definition of the source and structural models. With these input data we model the ground motion using the mode-coupling approach for sharply varying laterally heterogeneous anelastic media, i.e. computing the coupling coefficients for the modes transmitted and reflected at the vertical interface, between two quarter spaces in welded contact. The formalism can be readily applied to any laterally heterogeneous structure by using a custom series of layered anelastic structures in welded contact at vertical interfaces. The case of seismic wave propagation in smooth varying laterally heterogeneous layered media, is solved with an approximation, equivalent to WKBJ method. The theoretical formulation that combines both WKBJ and the coupling coefficients approaches, is implemented in a computer programs package. The computer code we have developed allows us to calculate synthetic seismograms for a wide range of laterally inhomogeneous layered anelastic media. As a case study we use an earthquake which occurred on December 16, 1999 (m_b=4.8) beneath the Bartlett sea trough, south of Santiago de Cuba city, that was recorded by an accelerometer (SMA-100) placed in Río Carpintero (RCC) station, at an epicentral distance of about 30 Km. The path travelled by the waves corresponds to a complex structure from undersea source to inland seismic station. A good fit between the observed transverse component (SH-waves) of acceleration and the corresponding synthetic signal is obtained for a seismic moment of M_0=7,1.10^{22}.

Introduction

The theory of surface wave propagation for several cases of laterally heterogeneous media has been developed since a long time (e.g. Levshin, 1985), but the practical implementation of computer codes is still in progress. The basic modal summation techniques, {e.g. Panza (1985), Panza and Suhadolc (1987), Florsch et al. (1991) and Panza et al. (2000)}, usually consider a laterally homogeneous layered elastic half-space, i.e. the classical 1D model. The problem of two laterally homogeneous (horizontally stratified) anelastic media in welded contact has been studied by several authors, and among them the modal summation implementations developed by Vaccari et al. (1989) and Romanelli et al. (1996, 1997) may be mentioned . The case of smooth varying laterally heterogeneous media, has been approximated by a combination of several laterally homogeneous media, not very different from the immediate neighbours in welded contact (Romanelli and Vaccari, 1999).

In the case of Southern Cuba, moderate and large size earthquakes are expected to occur beneath a deep sea trough, the Bartlett one (6 – 7 Km deep), and their main effects has to be expected in sites placed inland, which in the case of Santiago de Cuba city correspond to distances ranging from 30 to 100 Km. For a realistic site-specific seismic hazard assessment the propagation path has to consider the Oriente fault. In such a case, it is appropriate to consider a path formed by a smooth varying laterally heterogeneous medium from the source to the Oriente fault, in welded contact with a laterally homogeneous one from Oriente fault to the site. For this particular problem, based on the original formulation of Levshin (1985), an algorithm for solving the smooth laterally heterogeneous media problem has been developed .

Theory

One Dimensional case

The classical 1D problem of modes propagation, i.e. Rayleigh (P-SV motion) and Love (SH motion), has been considered by many authors (e.g. see Aki and Richards, 1980, for a review). The formalism of the complete synthesis of seismic signals by modal summation technique, including the attenuation due to anelasticity, can be found for example in Panza (1985), Panza and Suhadolc (1987) Florsch et al. (1991) and Panza et al (2000).

Following the notation given by Levshin (1985), the displacement spectra in the far-field approximation is:

$$u_{kD}^{q}(\omega,r,\varphi,z)=\frac{\exp(-i\cdot\pi/4)}{\sqrt{8\pi\omega}}\bullet\frac{\exp[-i\cdot\omega\cdot r/C_{kD}(\omega)-\omega\cdot r\cdot\alpha_{kD}(\omega)]}{\sqrt{r}}\bullet$$

$$\bullet\left[\frac{W_{kD}(\omega,z)}{\sqrt{U_{kD}(\omega)\cdot I_{kD}(\omega)\cdot C_{kD}(\omega)}}\right]\bullet\left[\frac{\varepsilon_{kD}^{q}(\omega,z)}{\sqrt{U_{kD}(\omega)\cdot I_{kD}(\omega)}}\right] \quad (1)$$

Here, q indicates the component of motion, i.e. (r,φ,z) in a cylindrical reference system or (x,y,z) in a Cartesian orthogonal one, k indicates the number of the propagating mode, D indicates the kind of motion (D=R $\rightarrow$ Rayleigh, D=L $\rightarrow$ Love), and ε_{kD}^{q} is defined as:

$$\varepsilon_{kR}^{z}=1,\quad \varepsilon_{kR}^{r}=-i\cdot\chi_{k}(\omega,z),\quad \varepsilon_{kR}^{\varphi}=\varepsilon_{kL}^{z}=\varepsilon_{kL}^{r}=0,\quad \varepsilon_{kL}^{\varphi}=i \qquad (2)$$

where χ_k is the ellipticity of the k-th Rayleigh mode. The phase attenuation α is given by:

$$\alpha_{kD}(\omega)=\omega\cdot\left(2\cdot U_{kD}(\omega)\cdot Q_{kD}(\omega)\right)^{-1} \qquad (3)$$

where Q is the quality factor. The values of C (phase velocity), U (group velocity), I (energy integral), and α and χ are the solutions of the eigenvalues-eingenfunctions problem, in terms of frequency and of the mode considered. The term W_{kD} is a function of the moment tensor of the double-couple associated to the seismic source and of the eigenfunctions just mentioned:

$$W_{kD}(\omega,\varphi,z)=\sum_{j=1}^{6}M_j(\omega)\cdot G_{kD}^{j}(\omega,\varphi,z) \qquad (4)$$

where M_j represents the components of the moment tensor and G_{kD}^{j} are the Green function components for the 1D problem. Explicit formulae for them can be obtained in [Levshin, (1985) and Panza et al., (2000)]. This solution constitutes the basis for the development of the approaches to deal with laterally varying media that will be considered in the following. In the modal summation technique, the Fourier transform, U_D^q, of the synthetic seismogram is obtained by summing the spectra of the modes. Let $N(\omega)$ be that number, then:

$$U_D^q(\omega,r,\varphi,z)=\sum_{k=1}^{N(\omega)}u_{kD}^{q}(\omega,r,\varphi,z) \qquad (5)$$

In the following, by simplicity, we won't write the explicit ω-dependence (ω) of velocities, attenuation and energy integrals.

Two-dimensional case: coupling coefficients

The first problem that was considered corresponds to 2 laterally homogeneous media in welded contact (Herrera, 1964; Alsop, 1966; Alsop et al., 1974). Several authors developed it further, and the simplest formulation is presented by Levshin (1985). This formulation was slightly modified in order to include the anelastic attenuation, and a complete computer implementation was developed by Vaccari et al. (1989) and Romanelli et al. (1996, 1997). If the source is in the first medium (M_0), placed at a horizontal distance s_{M_0} from the boundary between them, and the receiver is in the second medium (M), at a distance s_M from it, (1) becomes, in the case of normal incidence:

$$u^{q}_{m_0 mD}(\omega,r,\varphi,z)=\frac{\exp(-i\cdot\pi/4)}{\sqrt{8\pi\omega}}\bullet\frac{\exp\left[-i\cdot\omega\cdot\left(s_{M_0}/C^{M_0}_{kD}+s_M/C^{M}_{kD}\right)-\omega\cdot\left(s_{M_0}\cdot\alpha^{M_0}_{kD}+s_M\cdot\alpha^{M}_{kD}\right)\right]}{\sqrt{\left.J_{m_0 mD}(\omega)\right|_{M_0 M}}}\bullet$$

$$\bullet\,\gamma^{m_0 m}_{DP}\bullet\left[\frac{W_{kD}(\omega,z)}{\sqrt{U_{kD}\cdot I_{kD}\cdot C_{kD}}}\right]_{M_0}\bullet\left[\frac{\varepsilon^{q}_{kD}(\omega,z)}{\sqrt{U_{kD}\cdot I_{kD}}}\right]_{M}\qquad(6)$$

Here the displacement spectra is not evaluated for a mode k as before, but it is calculated for the mode m of the structure in which the receiver is placed, coupled with a mode m_0 of the structure in which the source is located. In principle, each one of the $N_{M_0}(\omega)$ modes of the source structure can be coupled with all the $N_M(\omega)$ modes of the receiver structure. Thus, the maximum number of coupled modes that could arrive to the receiver is $N_{M_0}(\omega)\cdot N_M(\omega)$. The term $\gamma^{m_0 m}_{DP}$ represents the coupling coefficient, whose value can vary in a range between 0 and 1 and it corresponds to transmitted waves (P=T) or to reflected waves (P=R). Its general form is

$$\gamma^{m_0 m}_{DP}=\frac{\left\langle A^{m_0}_{DP},A^{m'(P)}_{DM'}\right\rangle}{\left\langle A^{m_0}_{DM_0},A^{m_0}_{DM_0}\right\rangle^{1/2}\cdot\left\langle A^{m'(P)}_{DM'(P)},A^{m'(P)}_{DM'(P)}\right\rangle^{1/2}}\quad,\quad m'(P),M'(P)=\begin{Bmatrix}m_0,M_0\Rightarrow P=R\\ m,M\Rightarrow P=T\end{Bmatrix}\qquad(7a)$$

where the coefficients A^{m}_{DP} are the stress-displacement vectors associated to a particular mode in a specific medium. The numerator of this formula represents an integral that corresponds to a scalar product, defined as:

$$\left\langle A^{m_i}_{i},A^{m_j}_{j}\right\rangle=\frac{1}{2i}\int_0^\infty\left[\overrightarrow{u^{m_i}_{i}}\circ\left(\overleftrightarrow{\overset{\approx}{\sigma^{m_j}_{j}}}\circ\overrightarrow{e_u}\right)-\left(\overleftrightarrow{\sigma^{m_i}_{i}}\circ\overrightarrow{e_u}\right)\circ\overrightarrow{\overset{\approx}{u^{m_j}_{j}}}\right]dz\qquad(7b)$$

The symbol "≈" over a variable means complex conjugate, the $\overrightarrow{u_k^{m_k}}$ are the displacement spectra vectors. The $\overleftrightarrow{\sigma_k^{m_k}}$ are the dyadics that represent the stress, and $\overrightarrow{e_u}$ is a unitary vector in the direction of the wave propagation. The open dots mean dot product. The analytical solutions of this integral can be found in Vaccari et al. (1989) and Romanelli et al. (1997) for Rayleigh modes and in Romanelli et al. (1996) for Love modes. The stress-displacement vectors of transmitted (P=T) and reflected waves (P=R) are calculated by the application of Snell's law to the vertical boundary, in the form:

$$A_{kT}^{m_0} = (1-\xi(z)) \cdot A_{kM_0}^{m_0}, \quad A_{kR}^{m_0} = \xi(z) \cdot A_{kM_0}^{m_0} \qquad (8)$$

where ξ is the reflection coefficient.

The geometrical spreading J, for the transmitted wave, is:

$$J_{kD}(\omega) = \frac{1}{C_{kD}^{M_0}} \cdot \left(s_{M_0} \cdot C_{kD}^{M_0} + s_M \cdot C_{kD}^{M}\right) \qquad (9)$$

Two-dimensional case: WKBJ aproximation

The case of mode propagation in smooth varying laterally heterogeneous layered media was first solved by Woodhouse (1974). He presented an approximation, equivalent to the WKBJ method, that requires the use of variational principles, perturbation theory, propagator matrices and method of characteristics. This approach was further developed by other authors, and a complete treatment of the problem, in terms of free normal modes, can be found in Dahlen and Tromp (1998). A different WKBJ procedure was introduced Babich et al. (1976). In this approach, which follows the classical ray theory (Babich, 1960), the solution is sought in terms of waves propagating with velocity $c = |\nabla\tau|^{-1}$ along a ray (where τ is the wave front surface), by means of asymptotic methods. The conditions of smooth lateral variation can be summarised as follows. Let ρ_j, α_j and β_j be the density, P and S waves velocities respectively, in the j-th layer. They can vary into the layer only in a small range, i.e.:

$$\rho_j = \rho_j(\varepsilon x, \varepsilon y, z), \quad \alpha_j = \alpha_j(\varepsilon x, \varepsilon y, z), \quad \beta_j = \beta_j(\varepsilon x, \varepsilon y, z), \quad z_j \le z \le z_{j+1}, \quad j = 0,1,\ldots\ldots,N \quad (10a)$$

and the boundaries between layers are described by $z_j = z_j(\varepsilon x, \varepsilon y)$, where ε is a small parameter. The smoothness condition is:

$$|\nabla_\perp \alpha_j| \cdot \lambda_{max} / \alpha_j \langle\langle 2\pi, \quad |\nabla_\perp \beta_j| \cdot \lambda_{max} / \beta_j \langle\langle 2\pi, \quad |\nabla_\perp \rho_j| \cdot \lambda_{max} / \rho_j \langle\langle 2\pi, \quad |\nabla_\perp z_j| \cdot \lambda_{max} / z_j \langle\langle 2\pi \quad (10b)$$

where $\nabla_\perp$ is the horizontal gradient, and λ_{max} is the largest considered wavelength. The general solution is presented by Levshin (1985), and is discussed in detail in (A.A.V.V., 1989). It has been slightly modified by us to include attenuation, i.e.:

$$u^q_{kD}(\omega, r, \varphi, z) = \frac{\exp(-i \cdot \pi / 4)}{\sqrt{8\pi\omega}} \bullet \frac{\exp\left[-i \cdot \omega \int_{Mo}^{M} ds / C_{kD} - \omega \cdot \int_{Mo}^{M} \alpha_{kD} \cdot ds \right]}{\sqrt{J_{kD}(\omega)\big|_{M_0 M}}} \bullet$$

$$\bullet \left[\frac{\varepsilon^q_{kD}(\omega, z)}{\sqrt{U_{kD} \cdot I_{kD}}} \right]_M \bullet \left[\frac{W_{kD}(\omega, z)}{\sqrt{U_{kD} \cdot I_{kD} \cdot C_{kD}}} \right]_{Mo} \qquad (11)$$

The geometrical spreading J, in its general form can be defined as:

$$J = \left[\left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2 \right]^{1/2} \qquad (12)$$

The condition η=const. defines the ray path and x,y are the Cartesian orthogonal coordinates in a plane normal to the ray. For the special case of rays travelling without changes in horizontal direction, the value of J at a point M of the path can be derived from the kinematic ray tracing equations as:

$$J_{kD}(\omega)\big|_M = \frac{1}{C^{M_0}_{kD}} \bullet \int_{Mo}^{M} C_{kD} \cdot ds \qquad (13)$$

where $C^{M_0}_{kD}$ corresponds to the phase velocity at the source.

The four factors in the right-term of (11) can be interpreted as follows. The first is general, and it appears in any surface wave formulation; the second represents the trajectory, while the third and the fourth represent the receiver and the source contributions, which are evaluated by considering the solution of the 1D problem in the source and in the receiver models, respectively.

The two formulations are obtained from different mathematical procedures, i.e., coupling coefficients from analytical method and WKBJ from an asymptotic one. Nevertheless, the WKBJ formulae can be obtained from the coupling coefficients formulation through a limit procedure. Let us consider a medium formed by n 1D models in welded contact at n-1 boundaries. Let us assume also the case of no mode conversion, i.e., a mode of a given ordinal number can be transmitted only into a mode with the same ordinal number in the next structure. Such a problem has the solution:

$$u^q_{mD}(\omega,r,\varphi,z)=\frac{\exp(-i\cdot\pi/4)}{\sqrt{8\pi\omega}}\bullet\frac{\exp\left[-i\cdot\omega\cdot\left[\sum_{l=1}^{n}s_{M_l}/C^{M_l}_{kD}\right]-\omega\cdot\left(\sum_{l=1}^{n}s_{M_l}\cdot\alpha^{M_l}_{kD}\right)\right]}{\sqrt{J_{m_0D}(\omega)\Big|_{M_0M}}}\bullet$$

$$\bullet\prod_{j=1}^{n-1}\gamma^m_{j,D}\bullet\left[\frac{W_{kD}(\omega,z)}{\sqrt{U_{kD}\cdot I_{kD}\cdot C_{kD}}}\right]_{M_0}\bullet\left[\frac{\varepsilon^q_{kD}(\omega,z)}{\sqrt{U_{kD}\cdot I_{kD}}}\right]_{M} \qquad (14a)$$

where the index "l" runs over 1D media and the index "j" runs over the boundaries; the geometrical spreading is:

$$J_{kD}(\omega)=\frac{1}{C^{M_0}_{kD}}\cdot\left(\sum_{l=1}^{n}s_{M_l}\cdot C^{M_l}_{kD}\right) \qquad (14b)$$

A case study of this problem can be found in (Romanelli and Vaccari, 1999). If "n" tends to ∞, the horizontal extensions of the individual 1D models s_{M_l} become infinitesimal, and formula (14a) transforms into formulas (11,13) with the exception of the term in the $\gamma^m_{j,D}$. In the limit, these single coefficients have to be equal to (1,0), i.e. with unitary amplitude and with phase 0, and then, their product will also be equal to (1,0), The approximation used in the developed computational algorithm is consistent with this procedure.

Two-dimensional case: coupling coefficients and WKBJ approximation

Let us now consider a complex laterally heterogeneous layered anelastic medium composed by n_1 smooth varying laterally heterogeneous sections and n_2 laterally homogeneous ones, randomly disposed and in welded contact with each other through vertical surfaces (Fig.1). Levshin (1985) gives the general formula for the displacement spectra of the transmitted waves. Here we present a modified version that considers the anelastic attenuation, and that is limited to the normal incidence at the boundaries:

$$u^q_{kD}(\omega,r,\varphi,z)=\frac{\exp(-i\cdot\pi/4)}{\sqrt{8\pi\omega}}\bullet\left[\frac{W_{kD}(\omega,z)}{\sqrt{U_{kD}\cdot I_{kD}\cdot C_{kD}}}\right]_{M_0}\bullet\left[\frac{\varepsilon^q_{kD}(\omega,z)}{\sqrt{U_{kD}\cdot I_{kD}}}\right]_{M}\bullet\gamma_{kD}\bullet$$

$$\bullet\frac{\exp\left\{-i\cdot\omega\cdot\left[\sum_{j=1}^{n_1}\int_{l_j}\frac{ds}{C_{kD}}+\sum_{j=1}^{n_2}\frac{s_j}{C^j_{kD}}\right]\right\}-\exp\left\{\omega\cdot\left[\sum_{j=1}^{n_1}\int_{l_j}\alpha_{kD}\cdot ds+\sum_{j=1}^{n_2}s_j\cdot\alpha^j_{kD}\right]\right\}}{\sqrt{J_{kD}(\omega)}} \qquad (15)$$

The sum from j=1 to n_1 is for the smooth laterally varying sections, while the sum from j=1 to n_2 is for the laterally homogeneous ones. The s_j are the horizontal extensions of the different sections, the indexes D and q are the same as in previous formulas, with the exception of the index k that doesn't identify the mode, but the conversion from the source to receiver. More specifically, when waves cross a vertical boundary, each mode is coupled with all the modes of the next structure (Vaccari et al., 1989; Romanelli et al., 1996, 1997) in a way that a mode m in the structure of the source arrives to the receiver as a mode m', but having travelled the intermediate structures coupling with any of the N_j possible modes in any of the structures. Therefore, k means the mode conversion from source to receiver. In our case we have $n=n_1+n_2$ different structures and correspondingly n-1 vertical boundaries.

For structure j at frequency ω let $M_j(\omega)$ be the set of all these modes $M_j(\omega) = \{1, 2, \cdots\cdots, N_j(\omega)\}$ where $N_j(\omega)$ is the highest mode considered. The Cartesian product of the modes existing in each structure will represent the total possible combinations of mode conversions from source to receiver:

$$M_T(\omega) = M_1(\omega) \times M_2(\omega) \times \cdots\cdots \times M_{n-1}(\omega) \times M_n(\omega) \qquad (16a)$$

The total number of elements of the set $M_T(\omega)$ will be $N_T(\omega) = \prod_{j=1}^{n} N_j(\omega)$. Let k be one member of this set, corresponding to a particular mode conversion. Then, k is the n-tuple, defined as:

$$k = \left(m_1^k, \cdots\cdots, m_N^k\right) : m_1^k \in M_1, \cdots\cdots, m_n^k \in M_n \qquad (16b)$$

where m_j^k is the ordinal number of the mode in structure j in which the wave is travelling in the mode conversion k $[k = 1, 2 \ldots\ldots, N_T(\omega)]$. Going from one medium to the next, the amplitude and phase of the waves change in a way given by the coupling coefficients (Vaccari et al. 1989; Romanelli et al, 1996, 1997) $\gamma_{DP}^{mm'}$, where m means the mode incoming to the boundary and m' the outgoing one. In the case of several boundaries, the resulting amplitude and phase change will be the product of all the coupling coefficients (Panza et al., 2000). Then, the γ_{kD} of our formula will be:

$$\gamma_{kD} = \prod_{j=1}^{n-1} \gamma_D^{m_j^k m_{j+1}^k} \qquad (17)$$

where the $\gamma_D^{m_j^k m_{j+1}^k}$ are equivalent to the $\gamma_{DP}^{m_0 m}$ defined above, in which we dropped the index P because we are considering only the transmitted wave case. The evaluation of the geometrical spreading J is in general quite cumbersome, therefore we consider only the normal incidence case. Then:

$$J_{kD}(\omega) = \frac{1}{C_{0kD}} \cdot \left[\sum_{j=1}^{n_1} \int_{l_j} C_{kD} \cdot ds + \sum_{j=1}^{n_2} C_{kD}^{j} \cdot s_j \right] \qquad (18)$$

The sum over the modes can't be performed over mode numbers, but we have to sum over the n-tuples of the set $M_T(\omega)$:

$$U_D^q(\omega,\varphi,z) = \sum_{k=1}^{N_T} u_{kD}^q(\omega,\varphi,z) \qquad (19)$$

The algorithm

The practical implementation of equation (15) in a computer code is not simple. In fact, the complex medium described by formula (15), includes a combination of coupling coefficients (equations 6-9) and of WKBJ (equations 10-13) approximations. All the algorithms are based on the solution of the 1D problem (formulas 1-5), that exists in fthe orm of stable, accurate and efficient programs that are discussed in detail by Panza (1985), Panza and Suhadolc (1987), Florsch et al. (1991) and Panza et al. (2000). The algorithm for the coupling coefficient case has been discussed by Vaccari et al. (1989), Romanelli et al. (1996, 1997) and Romanelli and Vaccari, (1999). At any boundary between 1D structures, each mode of the incident wave field can be coupled with all the modes existing in the adjacent structure. This means that typical values of $N_T(\omega)$ can be as large as 200^n. Thus, on average, a matrix should be calculated with approximately 200^n rows, and with columns corresponding to frequency, coupling coefficient γ_{kD}, and n-tuple information given by formula (16b). This means that, when the numbers of vertical interfaces (n-1) are big, it is practically impossible to process the necessary files due to their size, and several procedures have to be used to reduce the data volume (e.g. Romanelli and Vaccari, 1999).

For the WKBJ approximation, the calculation of integrals is somewhat complicated, because of the conditions on the smoothness (formulas 10a,b). An exact solution requires the knowledge of the continuous functions describing each variable along the path, which is practically impossible, and it is necessary to make an approximation. This can be done by transforming the smooth inclined boundaries between layers in small steps "stairs" (Schwab, 1994). In Fig. 2a a section of a simple inclined boundary approximated by two horizontal segments shifted in vertical direction is shown. When this is done for the whole depth of the structure, and along the entire trajectory, the 2D resultant model is like the one shown in Fig. 2b. This modelling is controlled by the parameter δh, the step height. If we want to neglect reflected and diffracted waves, connected to the coupled transmitted modes, these steps have to be negligible for the propagation of involved waves. The condition to be satisfied is that $\lambda_{min}/\delta h>20$, where λ_{min} is the minimum involved wave length (Schwab, 1994). Therefore, a smooth laterally varying heterogeneous medium is substituted by a set of contiguous, laterally homogeneous ones that satisfy the condition $\lambda_{min}/\delta h>20$. It is supposed, and this is the basic assumption of the algorithm, that in passing from one to another of these "almost similar" lateral heterogeneous structures, each mode propagates without transformation in another mode. The complete solution of the problem requires the solution of the eigenvalue-eigenfunction problem in each of the 1D models. Then, the modal summation is performed using the "local modes" (Maupin, 1988) for each 1D structure, i.e. the modes that propagate through the smooth laterally varying media, correspond to the normal modes that would exist in each single structure as if it would be an infinite layered halfspace.

In our algorithm, the first step is to divide the smooth laterally varying heterogeneous medium in several zones, separated by vertical boundaries. Their position is defined by the condition that the interfaces between the layers change monotonically along the horizontal direction. Then these zones are subdivided again (if necessary) in sectors where these boundaries can be approximated by inclined right lines. Each sector is then automatically processed by interpolating horizontally layered homogeneous structures not very different one from each other ($\lambda_{min}/\delta h>20$ as suggested by Schwab, 1994). This results in a number of 1D structures that may be considerably large, depending on how the original sector changes (for instance, for the case f_{max}=1 Hz, the sector of inclined boundaries of Fig. 3, that shows small variations in the vertical position of boundaries, has been approximated by 51 1D structures). The spacing between interpolated structures may be uniform or controlled by the particular geometry of one selected layer.

In order to consider the entire problem it is necessary to analyse the case of the coupling between a 1D medium and a smooth laterally varying layered medium, or between two media of the last kind, that may be present along the waves path (see Fig. 1). The coupling has to be calculated for the last (or the first) of the interpolated 1D structures, taking into account that through a smooth laterally varying layered medium only those modes that exist in all the interpolated 1D structures can be propagated.

Data

Cuban Island is situated in the North American plate at its boundary with the Caribbean one. South of Eastern Cuba, this boundary is formed mainly by the Oriente fault shallow seismic zone, which is characterised by the presence of the Bartlett Sea trough, that is about 3 Km deep at a distance of 30 Km from the city. The most important earthquakes that affect Santiago de Cuba City occur beneath this trough, that reaches a maximum of 6-7 Km depth at a distance of 100 Km from the city. Recently, in Eastern Cuba a small accelerographic network composed by 4 Chinese SMA-100 triaxial accelerographs, with digital recording at 50/100 samples per second, 2-15 seconds of pre-event memory, and a flat response from 0 to 40 Hz has been installed. One of the accelerographs has been installed on bedrock at Río Carpintero seismic station, 18 Km to the east of Santiago de Cuba city. This accelerograph recorded the earthquake of Dec. 16, 1999, m_b=4.8 (no Ms was determined), h=17 Km (about 30 Km epicentral distance), that was felt with a maximum MSK intensity of V in the city. The focal region is under the Bartlett trough, in a place where the depth of the sea reaches 3-3.5 Km. In fig. 4 the geographical position of epicentre and the accelerographic station, and in Fig. 5 the recorded accelerograms are presented. The maximum recorded acceleration is 7.3 cm/sec^2, that is consistent with the intensity IV, felt in the seismic station (RCC observatory register).

Deep seismic sounding and other geophysical investigations provide data on the density and on the velocity of P waves until the Moho discontinuity. For the Bartlett trough the work of Ewing et al. (1960) has been used, while for the inland territory the results of Bovenko et al. (1980), as reinterpreted by Arriaza (1998), have been considered. For depths ranging from 30 to 150 Km the results of the tomographic P waves study of Van der Hilst (1990) and of the gravimetric study of Orihuela and Cuevas (1993) have been considered. For depths greater than 150 Km, the standard oceanic model of Harkrider (1970) is used. A set of 1D structural models has been constructed (Fig. 6), formed by 3 "sea" structures (S1, S2, S3) and 1 inland structure (L). They have been used to build the complex model assigned to the path hypocenter-recording station, whose uppermost 50 Km are shown in Fig. 7.

For calculating synthetic signals, it is necessary to know the seismic moment M_0 of the earthquake. We used two different procedures to pass from the measured magnitude, m_b, of the earthquake to the corresponding seismic moment. The first is through the use of one linear relationship of the kind M_S vs. m_b obtained by Garc˚ ìa (2001) using regional data from Cuba and surroundings, supplemented by the Kanamori (1977) M_w vs. $lg(M_o)$ formula, and the second by using the second order relationship $lg(M_0)$ vs. m_b, obtained by Johnston (1994) using global stable continental regions data. From the first we obtained the value M_0=2,9.10^{22} (M_W=4.25), and from the second the value M_0=1,7.10^{23} (M_W=4.75).

Since the available P-wave polarities do not allow us a reliable determination of the FPS, we decided to get a rough idea of the possible FPS using the ratio of the 3 observed components of motion, as follows. We considered, as an average model, the structure S3 of Fig. 5. With this structure we compute synthetic signals, with a maximum frequency of 1 Hz. The peak values of the 3 synthetic components of motion are compared with the corresponding ones obtained from the filtered (cut off 1 Hz) observed time series. The result of this trial and error procedure lead to the solution Az=270º, Dip=45º and Rake=45º, which is also in agreement with the distribution of the few reliable available polarities.

Results

The source receiver path, as can be seen in Fig. 6 is modelled by a 2D structure from the source to the Oriente fault and by a 1D structure from the Oriente fault to the seismic station RCC. The computation in the 2D structure is performed with the WKBJ method. The interpolation of intermediate 1D "short length" structures is made between the three structures of Fig. 6 (beginning (S1), middle (S2) and end (S3) of the path) following the described procedure. The whole 2D path is formed by 65 single "short length" 1D structures. The complete SH spectra up to f_{max}=1 Hz have been calculated for all of the 1D structures and the coefficients for all possible mode couplings have been calculated for the boundary between structures S3 and L. The synthetic seismogram has then been obtained by formula (15).

For comparison with the real seismogram the synthetic signal is scaled by using the scaling law of Gusev (1983), as reported by Aki (1987). As the original magnitude of the earthquake is in m_b scale, the fit real-synthetic signals was sought in the interval formed by the values of the two conversions made: M_0=(2,9.10^{22}, 1,7.10^{23}). A satisfactory fit was obtained for a value of M_0=7,1.10^{22} (that corresponds to M_W=4.5). A small time delay (about 0.3 sec.) is observed between synthetic and real signals, that falls into the limits of the error of the origin time determination (Fig.8).

Conclusions

The computer programs package algorithm for the treatment of the complete wavefield, i.e. the P-SV and SH waves, propagating in smooth varying laterally heterogeneous anelastic media (SVLHAM), by WKBJ approach has been implemented. This new algorithm is combined with other packages (previously developed) for the calculation of synthetic seismograms in 2D media formed by several 1D lateral homogeneous media (LHM) in welded contact. As a result, the algorithm is able to calculate synthetic signals in complex media formed by several SVLHAM and LHM sections.

The observed accelerogram of the earthquake which occurred on December 16, 1999 (m_b=4.8) at the Bartlett deep sea trough, and recorded in a seismic station placed inland, at an epicentral distance of 31.5 Km, has been used to check the described method against observations. The focal mechanism was grossly estimated by a 1D parametric study, since no reliable FPS or CMT solution could be obtained. A good fit between observed transverse component (SH-waves) accelerogram and synthetic signal is obtained for a seismic moment of M_0=7,1.10^{22} (that corresponds to M_W=4.5).

Acknowledgements

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Figure Captions

Fig. 1. General example of complex laterally heterogeneous layered media. They can be sections of horizontally layered homogeneous media (LHM) and smooth varying laterally heterogeneous media (SVLHM) randomly distributed along path.

Fig.2. Transformation of smooth varying heterogeneous media. a) Approximation of a section of an inclined boundary by a one step horizontal line, b) example of how a smooth laterally varying heterogeneous media is converted onto a set of horizontally layered homogeneous ones [modified from Schwab (1984)].

Fig. 3) Approximation of a sector with inclined line boundaries by a small-step "stairs", under criteria $\lambda_{min}/\delta h>20$. The horizontal scale is augmented approximately 2 times.

Fig. 4) Geographical frame. a) General position of the study area; b) Epicenter of December 16, 1999, earthquake (NEIC) and Río Carpintero station (RCC).

Fig. 5. Accelerographic record of December 16, 1999 in bedrock, Rio Carpintero (RCC) station. Original frequency content of the signals reaches 25 Hz.

Fig. 6. First 40 Km of the structural models used for preparation of complex structural profile epicenter-recording station: S1 – at the source S2 – intermediate, S3 - at the northern end of the Bartlett Trough and L - at the receiver. Down from this depth all the profile has a common structure.

Fig. 7. Scheme of the uppermost 50 Km corresponding to the path epicenter (solid star) – RCC station (solid triangle), where the scale used represents real proportions.

Fig. 8. Comparison of real and synthetic signals. Real signal was shifted back 0.29 sec to obtain the fit.

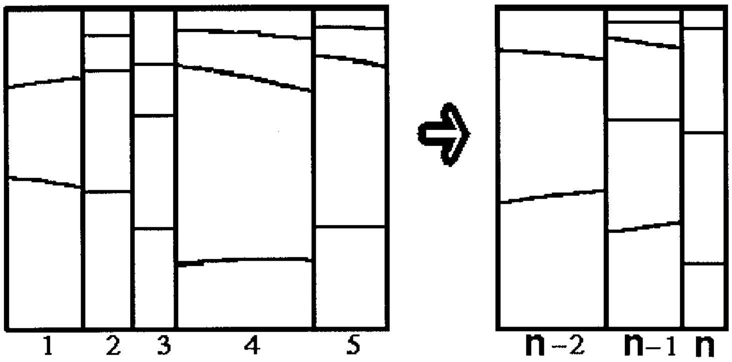


Fig. 1

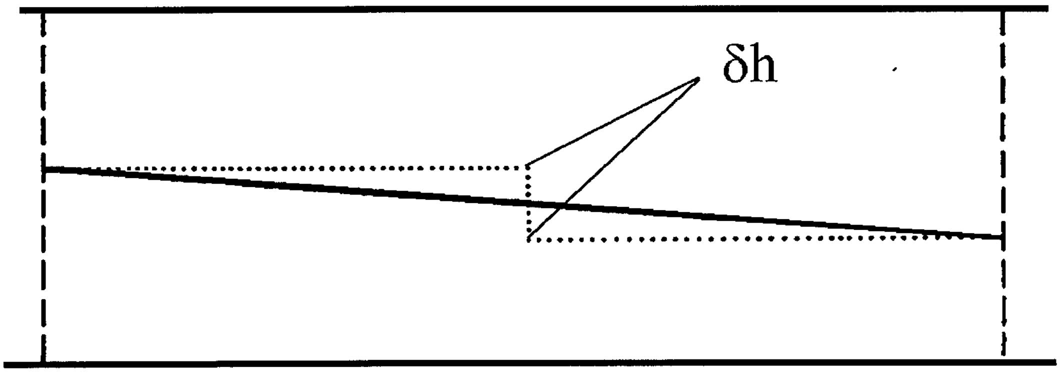


Fig. 2a

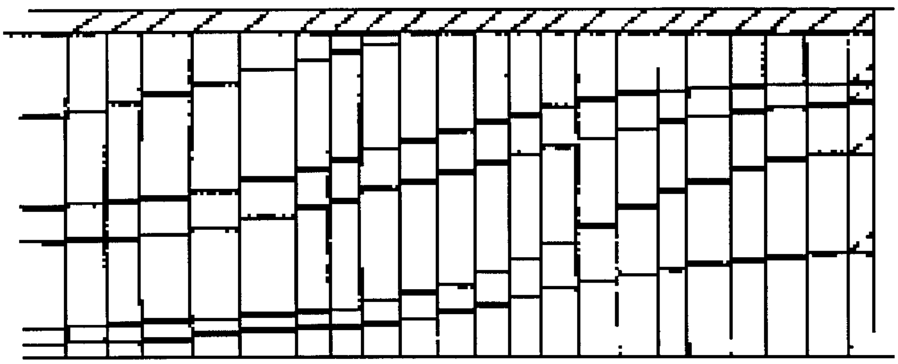


Fig. 2b

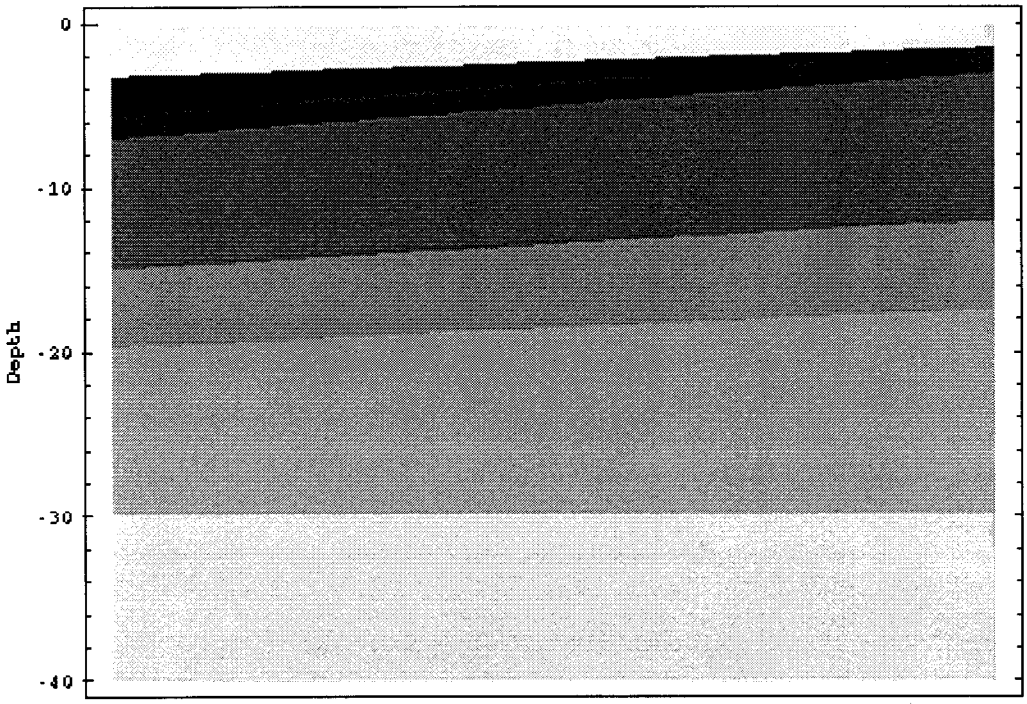


Fig. 3

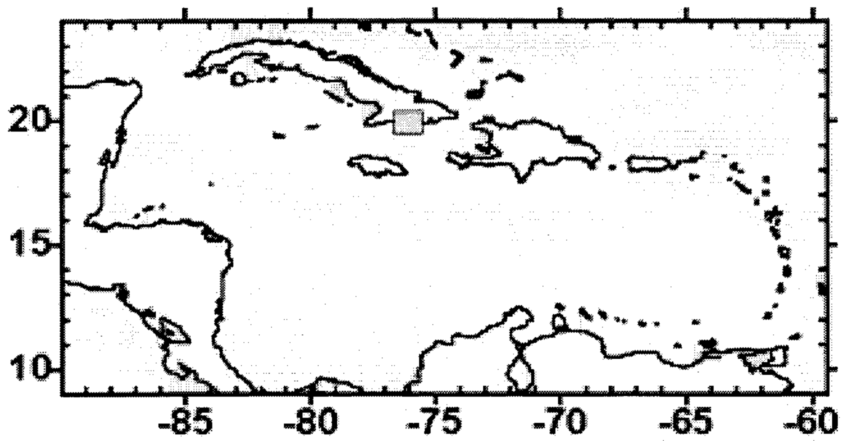


Fig. 4a

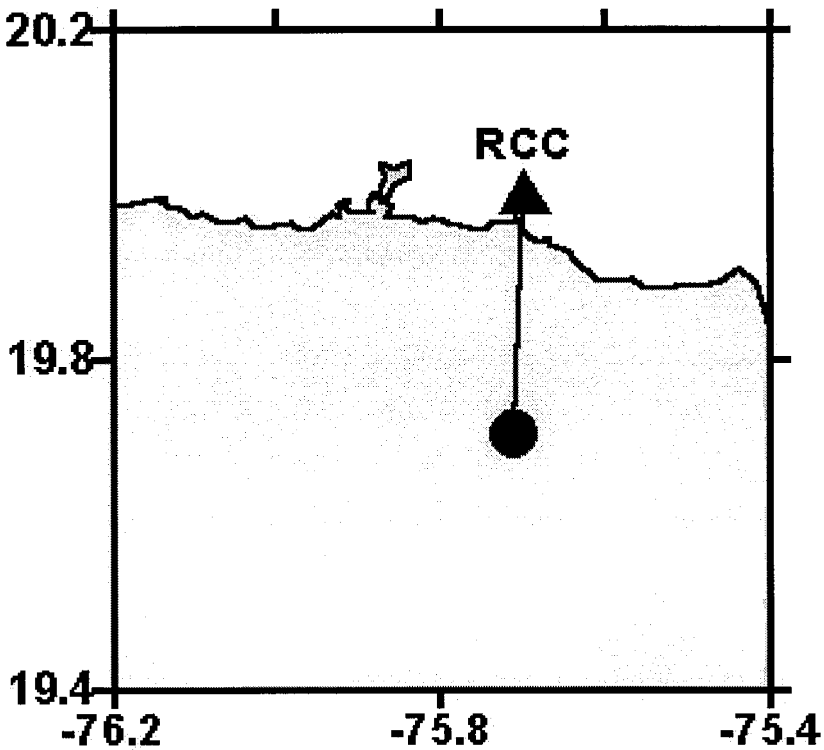
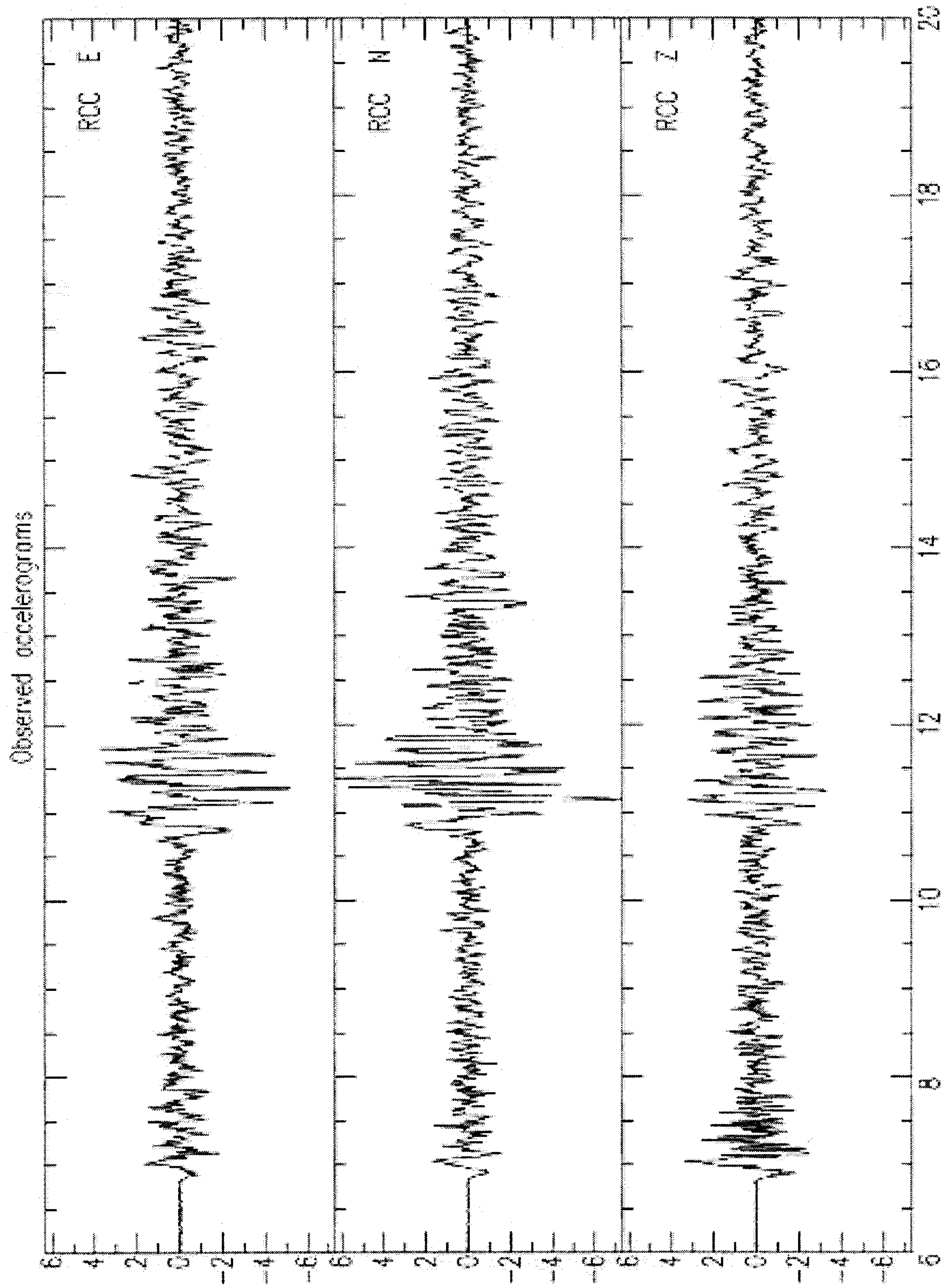


Fig. 4b



Observed accelerograms
RCC E
RCC N
RCC Z
6
4
2
0
-2
-4
-6
8
10
12
14
16
18
20

Fig. 5

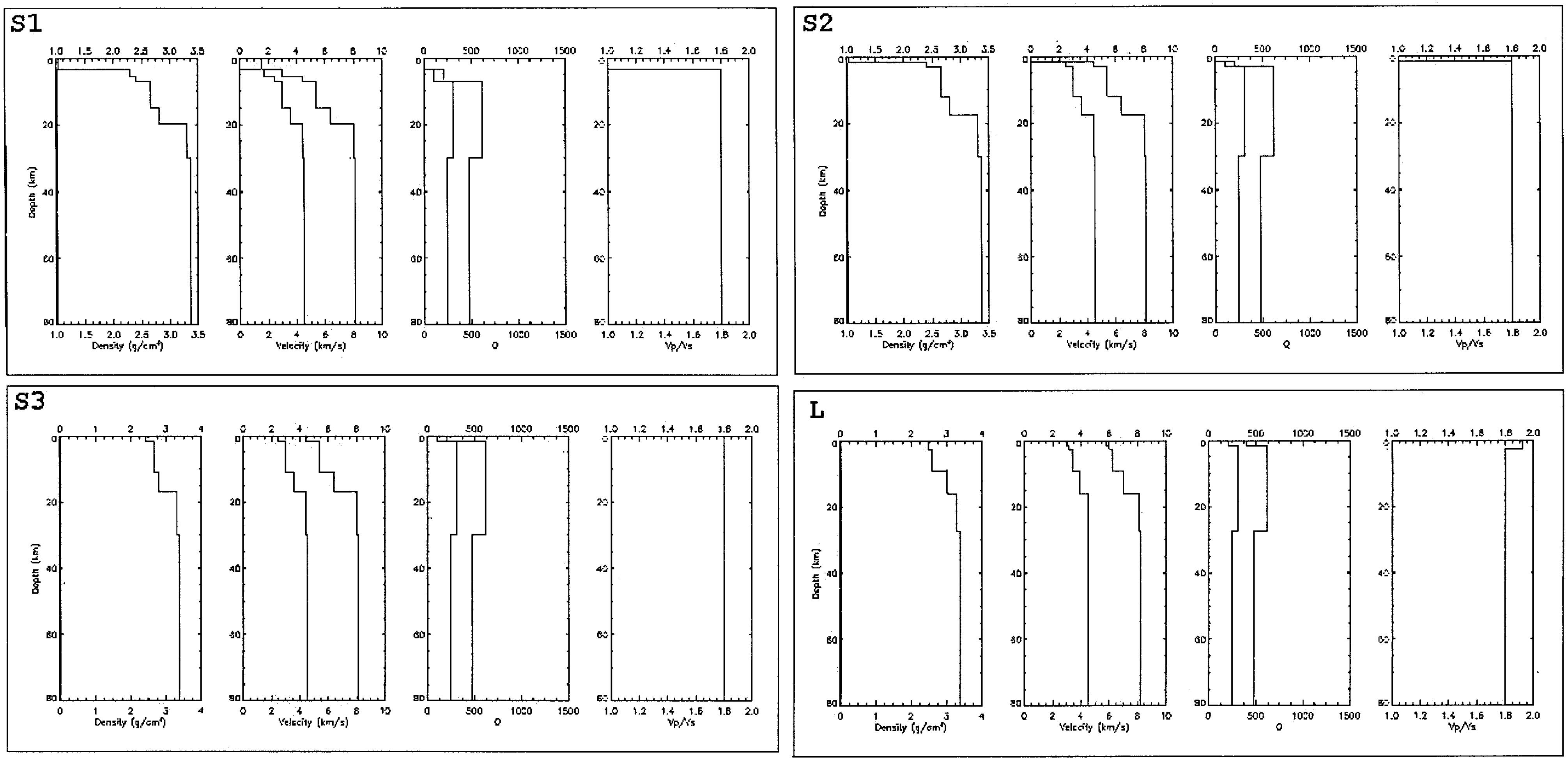


Fig. 6

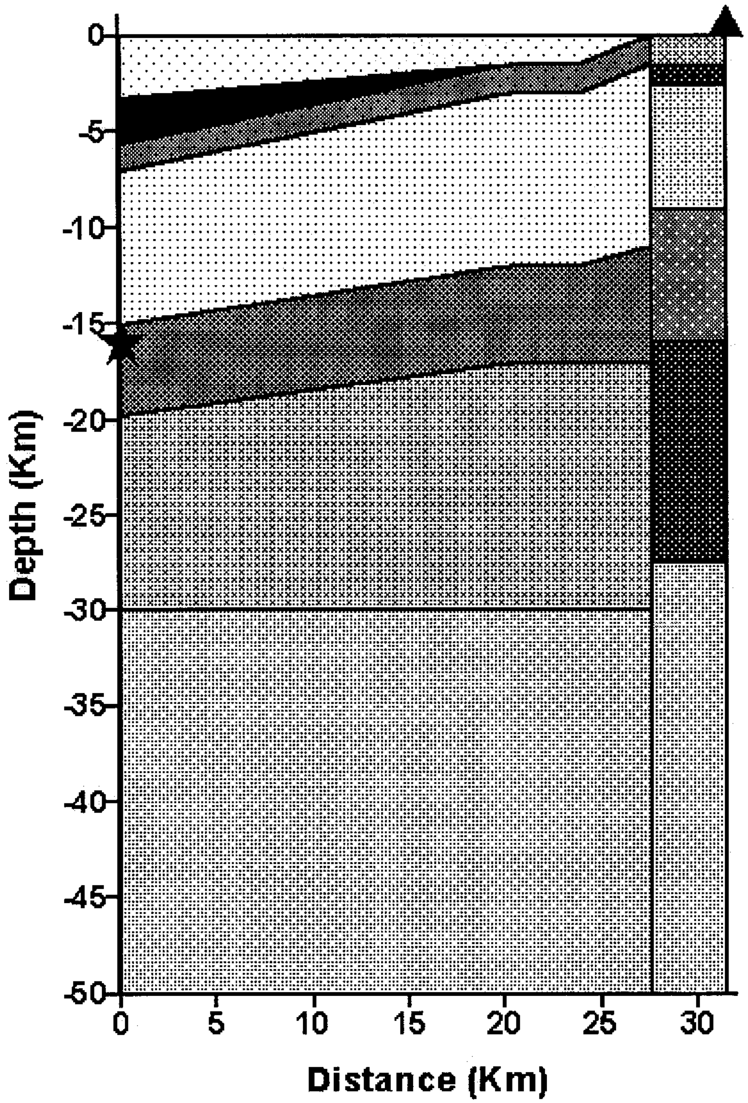

Depth (Km)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
0
5
10
15
20
25
30
Distance (Km)

Fig. 7

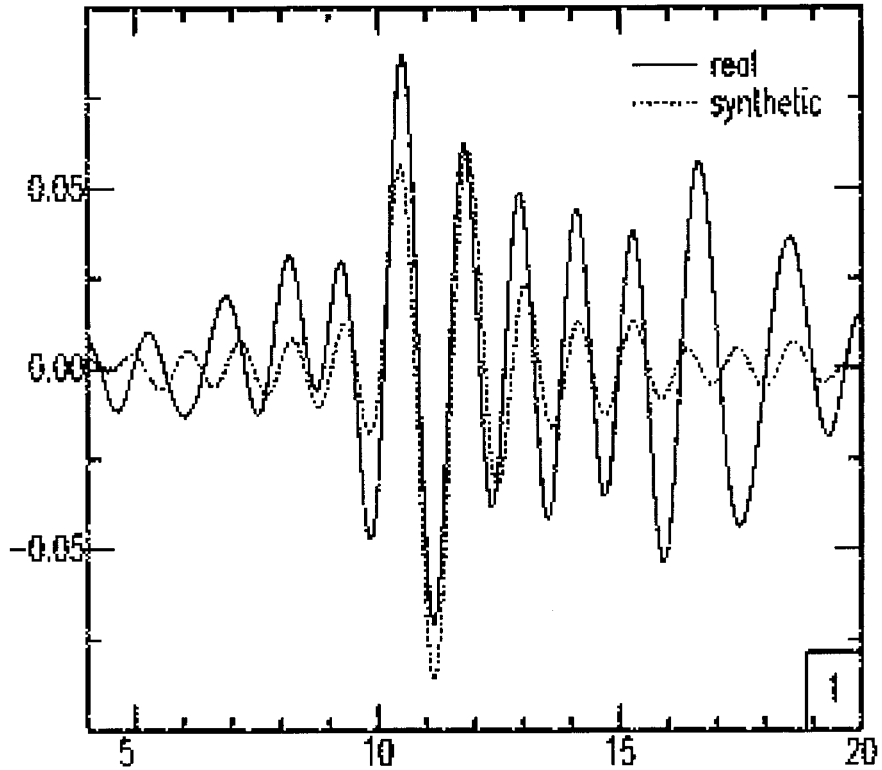

real
synthetic
0.05
0.00
-0.05
1
5
10
15
20

Fig. 8